Available online at www.sciencedirect.com

SCIENCE DIRECT®

International Journal of Solids and Structures 43 (2006) 887–898

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Nonlocal theory solution of two collinear cracks in the functionally graded materials

Zhen-Gong Zhou *, Biao Wang

Center for composite materials, Harbin Institute of Technology, P.O. Box 1247, Harbin 150001, PR China

Received 18 October 2004; received in revised form 5 April 2005
Available online 25 May 2005

Abstract

In this paper, the interaction of two collinear cracks in functionally graded materials subjected to a uniform anti-plane shear loading is investigated by means of nonlocal theory. The traditional concepts of the nonlocal theory are extended to solve the fracture problem of functionally graded materials. To make the analysis tractable, it is assumed that the shear modulus varies exponentially with the coordinate vertical to the crack. By use of the Fourier transform, the problem can be solved with the help of a pair of triple integral equations, in which the unknown variable is the displacement on the crack surfaces. To solve the triple integral equations, the displacement on the crack surfaces is expanded in a series of Jacobi polynomials. Unlike the classical elasticity solutions, it is found that no stress singularity is present near the crack tips. The nonlocal elastic solutions yield a finite hoop stress at the crack tip, thus allowing us to use the maximum stress as a fracture criterion in functionally graded materials. The magnitude of the finite stress field depends on the crack length, the distance between two cracks, the parameter describing the functionally graded materials and the lattice parameter of the materials.

Keywords: Collinear crack; Nonlocal theory; Functionally graded materials; Lattice parameter

1. Introduction

Functionally graded materials (FGMs) have been widely introduced and applied to the development of thermal and structural components due to its ability to not only reduce the residual and thermal stresses but to increase the bonding strength and toughness as well. To help the development of such materials, many analytical and theoretical studies in fracture mechanics have been widely done. In an attempt to address the

* Corresponding author. Tel.: +86 45 186412613; fax: +86 45 186418251.
E-mail address: zhouzhg@hit.edu.cn (Z.-G. Zhou).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2005.04.003

issues pertaining to the fracture analysis of bonded media with such transitional interfacial properties, a series of solutions to certain crack problems was obtained by Erdogan and his associates (Erdogan and Wu, 1997; Delale and Erdogan, 1988; Chen, 1990; Ozturk and Erdogan, 1996). Among them there are the solutions for a FGM strip containing an imbedded or an edge crack perpendicular to the surfaces (Erdogan and Wu, 1997); for a crack in the nonhomogeneous interlayer bounded by dissimilar homogeneous media (Delale and Erdogan, 1988); and for a crack at the interface between homogeneous and nonhomogeneous materials (Chen, 1990; Ozturk and Erdogan, 1996). Similar problems of delamination or an interface crack between a functionally graded coating and a substrate were considered in Jin and Batra (1996), Bao and Cai (1997) and Shbeeb and Binienda (1999). The crack problem in FGM layers under thermal stresses was studied by Erdogan and Wu (1996). They considered an unconstrained elastic layer under statically self-equilibrating thermal or residual stresses. However, it is found that all the solutions in Erdogan and Wu (1997), Delale and Erdogan (1988), Chen (1990), Ozturk and Erdogan (1996), Jin and Batra (1996), Bao and Cai (1997), Shbeeb and Binienda (1999) and Erdogan and Wu (1996) contain the stress singularity near the crack tips. This phenomenon is not reasonable according to the physical nature. As a result of this, beginning with Griffith, all fracture criteria in practice today are based on other considerations, e.g. energy, the *J*-integral (Rice, 1968) and the strain gradient theory (Xia and Hutchinson, 1996).

To overcome the stress singularity in the classical elastic fracture theory, Eringen (Eringen et al., 1977; Eringen, 1978, 1979) used nonlocal theory to discuss the stress near the tip of a sharp line crack in an isotropic elastic plate subject to uniform tension, shear and anti-plane shear, and the resulting solutions did not contain any stress singularities. This allows us to use the maximum stress as a fracture criterion. In contrast to these local approaches of zero-range internal interactions, the modern nonlocal continuum mechanics originated and developed in the last four decades. Edelen (1976), Eringen (1976), Green and Rivilin (1965) postulate that the local state at a point is influenced by the action of all particles of the body. According to nonlocal theory, the stress at a point X in a body depends not only on the strain at point X but also on that at all other points of the body. This is contrary to the classical theory that the stress at a point X in a body depends only on the strain at point X. In Pan and Takeda (1998), the basic theory of nonlocal elasticity was stated with emphasis on the difference between the nonlocal theory and classical continuum mechanics. The basic idea of nonlocal elasticity is to build a relationship between macroscopic mechanical quantities and microscopic physical quantities within the framework of continuum mechanics. The constitutive theory of nonlocal elasticity has been developed in Edelen (1976), in which the elastic modulus is influenced by the microstructure of the material. In Pan and Xing (1997) and Pan and Takeda (1997), it has been found that the microstructure of the material not only affects the constitutive equation, but also the basic balance laws and boundary conditions. Other results have been given by the application of nonlocal elasticity to the fields such as a dislocation near a crack (Pan, 1992, 1994), solid defects (Pan, 1996; Pan and Fang, 1996) and fracture mechanics problems (Pan, 1995; Pan and Fang, 1993). The literature on the fundamental aspects of nonlocal continuum mechanics is relatively extensive. The results of those concrete problems that were solved display a rather remarkable agreement with experimental evidence. This can be used to predict the cohesive stress for various materials and the results close to those obtained in atomic lattice dynamics (Eringen and Kim, 1974, 1977). Likewise, a nonlocal study of the secondary flow of viscous fluid in a pipe furnishes a streamline pattern similar to that obtained experimentally by Nikuradze (Eringen, 1977). Other examples of the effectiveness of the nonlocal approach are: (i) prediction of the dispersive character of elastic waves demonstrated experimentally (and lacking in the classical theory) (Eringen, 1972) and (ii) calculation of the velocity of short Love waves whose nonlocal estimates agree better with seismological observations than the local ones (Nowinski, 1984). Several nonlocal theories have been formulated to address strain-gradient and size effects (Forest, 1998). Recently, some fracture problems (Zhou et al., 1999, 2003; Zhou and Shen, 1999; Zhou and Wang, 2003; Sun and Zhou, 2004) in an isotropic elastic material and the piezoelectric material have been studied by use of nonlocal theory with a somewhat different method. However, relatively few works have been made for the fracture analysis by nonlocal theory in functionally graded materials due to the mathematical complexities. To our knowledge, the interaction of two collinear cracks in functionally graded materials has not been studied by use of the nonlocal theory. Thus, the present work is an attempt to fill this needed information. Here, we just attempt to give a theoretical solution for this problem.

In the present paper, the interaction of two collinear cracks subjected to anti-plane shear loading is investigated by use of nonlocal theory in functionally graded materials with Schmidt method (Morse and Feshbach, 1958; Itou, 1978; Yan, 1967). The Fourier transform is applied and a mixed boundary value problem is reduced to a pair of triple integral equations. To solve the triple integral equations, the displacement on the crack surfaces is expanded in a series of Jacobi polynomials. This process is quite different from those adopted in Erdogan and Wu (1997), Delale and Erdogan (1988), Chen (1990), Ozturk and Erdogan (1996), Jin and Batra (1996), Bao and Cai (1997), Shbeeb and Binienda (1999) and Erdogan and Wu (1996) as mentioned above. Numerical solutions are obtained for the stress field near the crack tip. Contrary to the previous results, it is found that the solution does not contain any stress singularities near the crack tip.

2. Basic equations of nonlocal theory

Basic equations of two-dimensional anti-plane shear plane of functionally graded materials, nonlocal elastic solid, with vanishing body force are

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0 \tag{1}$$

$$\tau_{xz}(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu^*(|x'-x|,|y'-y|)\frac{\partial w(x',y')}{\partial x'}\,\mathrm{d}x'\,\mathrm{d}y' \tag{2}$$

$$\tau_{yz}(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu^*(|x'-x|,|y'-y|)\frac{\partial w(x',y')}{\partial y'}\,\mathrm{d}x'\,\mathrm{d}y' \tag{3}$$

where $w(x,y)$ is the out-of-plane displacement. The only difference from the classical elasticity is in the stress constitutive equations (2) and (3), the stress τ_{xz} and τ_{yz}, at a point (x,y) depend on the $\frac{\partial w(x,y)}{\partial x}$ and $\frac{\partial w(x,y)}{\partial y}$, at all points of the body. For the nonhomogeneous materials, anti-plane shear problem there exits only a materials parameter $\mu^*(|x'-x|,|y'-y|)$, which is a function of the distance $d = \sqrt{(x'-x)^2+(y'-y)^2}$. In the papers of Eringen and Kim (1974, 1977) and Eringen (1977), we obtained the form of $\mu^*(|x'-x|,|y'-y|)$ for which the dispersion curves of plane elastic waves coincide with those known in lattice dynamics. Among several possible curves the following has been found to be very useful

$$\mu^*(|x'-x|,|y'-y|) = \alpha(|x'-x|,|y'-y|)\mu(y') \tag{4}$$

$$\alpha(|x'-x|,|y'-y|) = \alpha_0 \exp\{-(\beta/a)^2[(x'-x)^2+(y'-y)^2]\} \tag{5}$$

where $\alpha(|x'-x|,|y'-y|)$ is known as the influence function. β is a constant and can be obtained by an experiment. a is the characteristic length. The characteristic length may be selected according to the range and sensitivity of the physical phenomena. For instance, for the perfect crystals, a may be taken as the lattice parameter. For granular materials, a may be considered to be the average granular distance and for fiber composites, the fiber distance, etc. In the present paper, a is taken as the lattice parameter. $\mu(y)$ is the functionally graded material constant of the classical elasticity. α_0 is determined by the normalization

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha(|x'-x|,|y'-y|)\mathrm{d}x'\,\mathrm{d}y' = 1 \tag{6}$$

In the present work, we employ nonlocal elastic moduli given by Eqs. (4) and (5). Substituting Eqs. (4) and (5) into Eq. (6), it can be obtained, in a two-dimensional space

$$\alpha_0 = \frac{1}{\pi}(\beta/a)^2 \tag{7}$$

Crack problems in the functionally graded materials do not appear to be analytically tractable for arbitrary variations of material properties. Usually, one tries to generate the forms of nonhomogeneities for which the problem becomes tractable. Similar to the treatment of the crack problem for isotropic nonhomogeneous materials in Erdogan and Wu (1997), Delale and Erdogan (1988), Chen (1990) and Ozturk and Erdogan (1996), we assume the material properties are described by

$$\mu(y) = \mu_0 e^{\lambda y} \tag{8}$$

where μ_0 is the shear modulus of the functionally graded materials along $y = 0$.

Substituting Eqs. (2) and (3) into Eq. (1) and using Green–Gauss theorem leads to

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \alpha(|x'-x|,|y'-y|)\mu(y')\left[\frac{\partial^2 w(x',y')}{\partial x'^2} + \frac{\partial^2 w(x',y')}{\partial y'^2} + \lambda\frac{\partial w(x',y')}{\partial y'}\right]\mathrm{d}x'\,\mathrm{d}y' - \left[\int_{-1}^{-b} + \int_{b}^{1}\right]\alpha(|x'-x|,|y'-y|)[\sigma_{yz}(x',0^+) - \sigma_{yz}(x',0^-)]\mathrm{d}x' = 0 \tag{9}$$

where $\sigma_{yz}(x,y) = \mu(y)\frac{\partial w(x,y)}{\partial y}$. This expression is the classical Hooke's law. Here the surface integral may be dropped since the displacement field vanishes at infinity. $1 - b$ is the length of the crack as mentioned below.

3. The crack model

It is assumed that there are two collinear symmetric cracks of length $1 - b$ along the x-axis in functionally graded material plane as shown in Fig. 1. $2b$ is the distance between the two cracks (The solution of two collinear cracks of length $c - b$ in the piezoelectric materials can easily be obtained by a simple change in the numerical values of the present paper for crack length $1 - b/c$. $c > b > 0$.). The plate is subjected to a uniform constant anti-plane shear stress $\tau_{yz} = -\tau_0$ (τ_0 is a magnitude of the uniform anti-plane shear stress loading.) along the surfaces of the crack. As discussed in Eringen (1979), the boundary conditions can be written as follows:

$$\tau_{yz}(x,0^+) = \tau_{yz}(x,0^-) = -\tau_0, \quad b \leqslant |x| \leqslant 1, \ y = 0 \tag{10}$$

$$w(x,0^+) = w(x,0^-) = 0, \quad |x| > 1, \ |x| < b, \ y = 0 \tag{11}$$

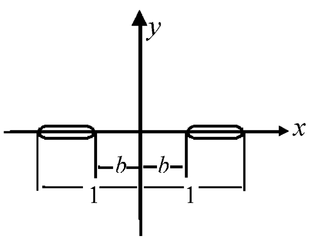

Fig. 1. Geometry and coordinate system for two collinear cracks.

4. The triple integral equation

As mentioned in Eringen (1979), it can be obtained that $[\sigma_{yz}(x,0^+) - \sigma_{yz}(x,0^-)] = 0$. So it can be shown that the general solution of (9) is identical to that of

$$\frac{\partial^2 w(x,y)}{\partial x^2} + \lambda \frac{\partial w(x,y)}{\partial y} + \frac{\partial^2 w(x,y)}{\partial y^2} = 0 \tag{12}$$

almost everywhere.

Because of the symmetry, it suffices to consider the problem for $x \geqslant 0$, $|y| < \infty$.

Eq. (12) can be solved giving

$$w(x,y) = \begin{cases} \frac{2}{\pi}\int_0^\infty A(s)\mathrm{e}^{-\gamma y}\cos(sx)\mathrm{d}s, & y \geqslant 0 \\ -\frac{2}{\pi}\int_0^\infty A(s)\mathrm{e}^{\gamma y}\cos(sx)\mathrm{d}s, & y \leqslant 0 \end{cases} \tag{13}$$

where $A(s)$ is an unknown function and $\gamma = \frac{\lambda + \sqrt{\lambda^2 + 4s^2}}{2}$.

Substituting (13) into (3), it can be obtained

$$\tau_{yz}(x,y) = -\frac{2\mu_0}{\pi}\int_0^\infty \gamma A(s)\mathrm{d}s \int_0^\infty \mathrm{e}^{-(\gamma-\lambda)y'}\mathrm{d}y' \int_{-\infty}^\infty [\alpha(|x'-x|,|y'-y|) + \alpha(|x'-x|,|y'+y|)] \times \cos(sx')\mathrm{d}x' \tag{14}$$

Substituting for α from (5), the integrations may be performed with respect to x' and y' by noting the integrals (Gradshteyn and Ryzhik, 1980)

$$\int_{-\infty}^\infty \exp(-px'^2)\begin{Bmatrix} \sin\xi(x'+x) \\ \cos\xi(x'+x) \end{Bmatrix}\mathrm{d}x' = (\pi/p)^{1/2}\exp\left(-\frac{\xi^2}{4p}\right)\begin{Bmatrix} \sin(\xi x) \\ \cos(\xi x) \end{Bmatrix} \tag{15}$$

$$\int_0^\infty \exp(-py^2 - \gamma y)\mathrm{d}y = \frac{1}{2}(\pi/p)^{1/2}\exp(\gamma^2/4p)[1 - \Phi(\gamma/2\sqrt{p})] \tag{16}$$

$$\Phi(z) = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-t^2)\mathrm{d}t \tag{17}$$

Hence

$$\tau_{yz}(x,y) = -\frac{\mu_0}{\pi}\int_0^\infty \gamma g(s,y)A(s)\cos(sx)\mathrm{d}s \tag{18}$$

where

$$g(s,y) = \mathrm{e}^{-py^2}\left\{\mathrm{e}^{-\frac{s^2-(\gamma-2py-\lambda)^2}{4p}}\left[1 - \Phi\left(\frac{\gamma-2py-\lambda}{2\sqrt{p}}\right)\right] + \mathrm{e}^{-\frac{s^2-(\gamma+2py-\lambda)^2}{4p}}\left[1 - \Phi\left(\frac{\gamma+2py-\lambda}{2\sqrt{p}}\right)\right]\right\}, \quad p = \left(\frac{\beta}{a}\right)^2.$$

So the boundary conditions (10) and (11) can be expressed as

$$\tau_{yz}(x,0) = -\frac{2\mu_0}{\pi}\int_0^\infty \gamma g_0(s)A(s)\cos(sx)\mathrm{d}s = -\tau_0, \quad b \leqslant x \leqslant 1 \tag{19}$$

$$\int_0^\infty A(s)\cos(sx)\mathrm{d}s = 0, \quad x > 1, \quad x < b \tag{20}$$

where

$$g_0(s) = e^{-\frac{s^2-(\gamma-\lambda)^2}{4p}}\left[1-\Phi\left(\frac{\gamma-\lambda}{2\sqrt{p}}\right)\right]$$

It can be obtained that $\mathrm{Lim}_{a\to 0}\, g_0(s) = 1$. So Eqs. (19) and (20) will revert to the well-known triple integral equations of the classical theory for the limit $a \to 0$. To determine the unknown function $A(s)$, the previous pair of triple integral equations (19) and (20) must be solved.

5. Solution of the triple integral equation

The only difference between the classical and nonlocal equations is in the influence function $g_0(s)$, it is logical to utilize the classical solution to convert the system Eqs. (19) and (20) to an integral equation of the second kind, which is generally better behaved. For the lattice parameter $a \to 0$, then $g_0(s)$ equals to a nonzero constant and Eqs. (19) and (20) reduce to a pair of triple integral equations for the same problem in classical elasticity. As discussed in Eringen et al. (1977), the triple integral equations (19) and (20) cannot be transformed into a Fredholm integral equation of the second kind, because $g_0(s)$ does not tend to a constant C ($C \neq 0$) for $s \to \infty$. Of course, the triple equations (19) and (20) can be considered to be a single integral equation of the first kind with discontinuous kernel. It is well-known in the literature that integral equations of the first kind are generally ill-posed in sense of Hadamard, i.e. small perturbations of the data can yield arbitrarily large changes in the solution. This makes the numerical solution of such equations quite difficult. To overcome the difficulty, the Schmidt method (Morse and Feshbach, 1958; Yan, 1967) is used to solve the triple integral equations (19) and (20). The displacement w on the crack surface can be represented by the following series:

$$w(x,0) = \sum_{n=0}^{\infty} b_n P_n^{(\frac{1}{2},\frac{1}{2})}\left(\frac{x-\frac{1+b}{2}}{\frac{1-b}{2}}\right)\left(1-\frac{\left(x-\frac{1+b}{2}\right)^2}{\left(\frac{1-b}{2}\right)^2}\right)^{\frac{1}{2}}, \quad \text{for } b \leqslant x \leqslant 1 \tag{21}$$

$$w(x,0) = 0, \quad \text{for } x > 1, \quad 0 < x < b \tag{22}$$

where a_n are unknown coefficients,$P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial (Gradshteyn and Ryzhik, 1980). The Fourier transform of (21) and (22) are (Erdelyi, 1954)

$$A(s) = \bar{w}(s,0) = \sum_{n=0}^{\infty} b_n F_n G_n(s) \frac{1}{s} J_{n+1}\left(s\frac{1-b}{2}\right) \tag{23}$$

$$F_n = 2\sqrt{\pi}\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{n!}, \quad G_n(s) = \begin{cases} (-1)^{\frac{n}{2}}\cos\left(s\frac{1+b}{2}\right), & n = 0,2,4,6,\ldots \\ (-1)^{\frac{n+1}{2}}\sin\left(s\frac{1+b}{2}\right), & n = 1,3,5,7,\ldots \end{cases} \tag{24}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively. A superposed bar indicates the Fourier cosine transform through the paper. The Fourier cosine transform is defined as follows:

$$\bar{f}(s) = \int_0^{\infty} f(x)\cos(sx)\mathrm{d}x, \quad f(x) = \frac{2}{\pi}\int_0^{\infty}\bar{f}(s)\cos(sx)\mathrm{d}s$$

Substituting (23) into Eqs. (19) and (20), it can be shown that Eq. (20) is automatically satisfied. Eq. (19) reduces to

$$\frac{2\mu_0}{\pi}\sum_{n=0}^{\infty} a_n F_n \int_0^{\infty} \frac{\gamma}{s} g_0(s) G_n(s) J_{n+1}\left(s\frac{1-b}{2}\right)\cos(sx)\mathrm{d}s = \tau_0, \quad b \leqslant x \leqslant 1 \tag{25}$$

From the relation

$$1 - \Phi(z) = \frac{e^{-z^2}}{\sqrt{\pi}z}\left[1 + \sum_{k=1}^{\infty}(-1)^k \frac{(2k-1)!!}{(2z^2)^k}\right]$$

it can be obtained that

$$e^{-\frac{s^2-(\gamma-\lambda)^2}{4p}}\left[1 - \Phi\left(\frac{\gamma-\lambda}{2\sqrt{p}}\right)\right] = \frac{2\sqrt{p}e^{-\frac{s^2}{4p}}}{\sqrt{\pi}(\gamma-\lambda)}\left[1 + \sum_{k=1}^{\infty}(-1)^k \frac{(2k-1)!!}{\left[2\left(\frac{\gamma-\lambda}{2\sqrt{p}}\right)^2\right]^k}\right] \tag{26}$$

The semi-infinite integral in Eq. (25) can be numerically evaluated directly by use of Eq. (26). So the semi-infinite integral in Eq. (25) can be evaluated numerically by Filon's method (Amemiya and Taguchi, 1969). Thus Eq. (25) can be solved for coefficients a_n by the Schmidt method (Morse and Feshbach, 1958; Yan, 1967). Here, it was omitted. But can be seen in Zhou et al. (1999, 2003), Zhou and Shen (1999) and Zhou and Wang (2003).

6. Numerical calculations and discussion

The coefficients a_n are known, so that the entire stress field can be obtained. However, in fracture mechanics, it is important to determine the stress τ_{yz} in the vicinity of the crack tips. In the case of the present study, τ_{yz} along the crack line can be expressed as

$$\tau_{yz}(x,0) = -\frac{2\mu_0}{\pi}\sum_{n=0}^{\infty} a_n F_n \int_0^{\infty} \frac{\gamma}{s} g_0(s) G_n(s) J_{n+1}\left(s\frac{1-b}{2}\right)\cos(sx)\,ds \tag{27}$$

When the lattice parameter $a \neq 0$, the semi-infinite integration and the series in Eq. (27) are convergent for any variable x, it gives a finite stress all along $y = 0$, so there is no stress singularity at crack tips. At $b < x < 1$, $\tau_{yz}/(-\tau_0)$ is very close to unity, and for $x > 1$, $\tau_{yz}/(-\tau_0)$ possesses finite values diminishing from a finite value at $x = 1$ to zero at $x = \infty$. Since $a/\beta l > 1/100$ represents a crack length of less than 100 atomic distances (Eringen et al., 1977), and for such submicroscopic sizes, other serious questions arise regarding the interatomic arrangements and force laws, we do not pursue solutions valid at such small crack sizes. The semi-infinite numerical integrals, which occur, are evaluated easily by Filon and Simpson's (Amemiya and Taguchi, 1969) methods because of the rapid diminution of the integrands. From Zhou et al. (1999, 2003), Zhou and Shen (1999), Zhou and Wang (2003) and Itou (2001), it can be seen that the Schmidt method is performed satisfactorily if the first ten terms of infinite series to Eq. (27) are retained.

The results are plotted in Figs. 2–7. The following observations are very significant:

(i) In the present paper, the traditional concepts of the nonlocal theory are extended to solve the fracture problem of functionally graded materials. The effects of the functionally graded parameter and the lattice parameter of the functionally graded materials upon the stress fields near the crack tip are considered by use of the nonlocal theory.

(ii) The nonlocal elastic solutions in functionally graded materials yield a finite hoop stress at the crack tip, thus allowing us to use maximum stress as a fracture criterion for the functionally graded materials. The maximum stress value does not occur at the crack tip, but slightly away from it as shown in Figs. 2–4. This phenomenon has been thoroughly substantiated by Eringen (1983). The distance between the crack tip and the maximum stress point is very small, and it depends on the crack length, the material properties and the lattice parameter. Contrary to the classical elasticity solution, it is

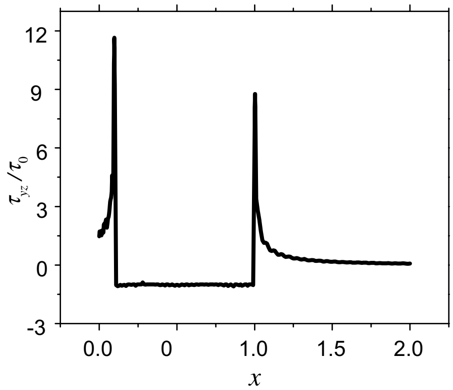

Fig. 2. The stress along the crack line versus x for $b = 0.1$, $a/\beta = 0.001$ and $\lambda = 0.2$.

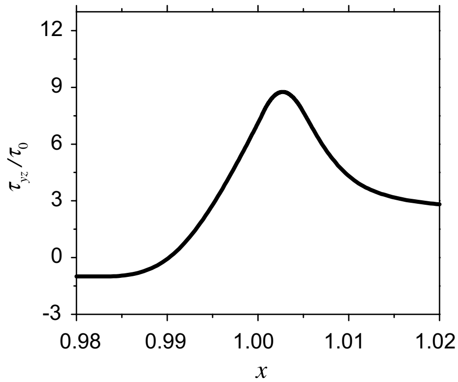

Fig. 3. The local enlarged graph of Fig. 2 near the crack right tip.

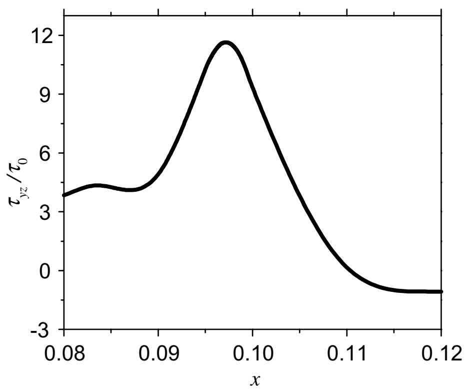

Fig. 4. The local enlarged graph of Fig. 2 near the crack left tip.

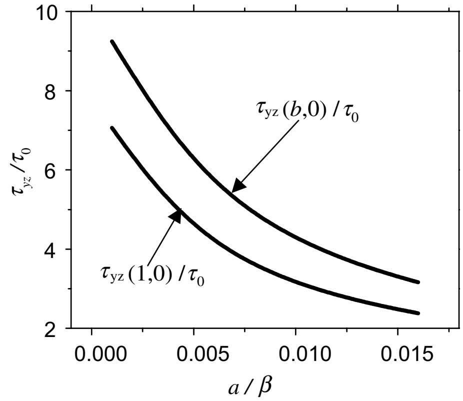

Fig. 5. The stress at the crack tip versus a/β for $b = 0.1$ and $\lambda = 0.2$.

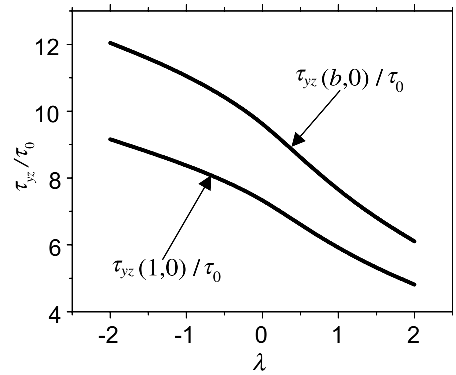

Fig. 6. The stress at the crack tip versus λ for $b = 0.1$ and $a/\beta = 0.001$.

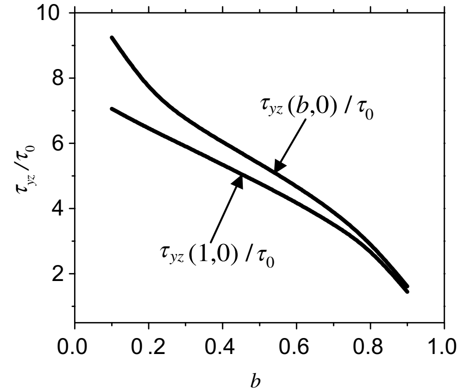

Fig. 7. The stress at the crack tip versus b for $\lambda = 0.2$ and $a/\beta = 0.001$.

found that the present results converge to the classical ones when far away from the crack tip as shown in Figs. 2–4. Simultaneously, for the nonlocal solution, the smaller the lattice parameter is, the more closer to the classical solution as shown in Figs. 2–4. This conclusion is the same as one in Eringen et al. (1977) and Eringen (1978, 1979) for the homogeneous materials.

(iii) The stress of τ_{yz} does not depend on the shear modulus μ_0 as shown in Eqs. (25) and (27). However, the stress of τ_{yz} depends on the crack length, the parameter describing the functionally graded materials and the lattice parameter of the functionally graded materials. This is the same as the anti-plane shear fracture problem in the isotropic homogeneous materials as shown in Eringen (1979).

(iv) The stress at the crack tip becomes infinite as the lattice parameter distance $a \to 0$. This is the classical continuum limit of square root singularity.

(v) The effect of the lattice parameter of the functionally graded materials on the stress field near the crack tips decreases with increase of the lattice parameter as shown in Fig. 5. This phenomenon is the same as one in Eringen (1979).

(vi) The effect of the parameter describing the functionally graded materials on the stress field near the crack tip decreases with increase of the parameter describing the functionally graded materials as shown in Fig. 6. This means that, by decreasing the gradient parameter of FGMs, the stress fields near the crack tips can be reduced.

(vii) The value of the stress τ_{yz} at the crack tip increases with increase of the crack length as shown in Fig. 7, i.e., the interaction of two collinear cracks increases with decrease of distance between two cracks. It can also be obtained that the crack left tip's stress fields are greater than the crack right tip's ones for the right crack as shown in Figs. 2–4 and 7.

(viii) The stress concentration occurs at the crack tips as stated by Eringen (1978, 1979), and this is given by

$$\tau_{yz}(1,0)/\tau_0 = c_{\mathrm{R}}/\sqrt{a/[2\beta(1-b)]} \tag{28}$$

$$\tau_{yz}(b,0)/\tau_0 = c_{\mathrm{L}}/\sqrt{a/[2\beta(1-b)]} \tag{29}$$

where c_{R} and c_{L} represent the stress concentration values at the right tip and at the left tip for the right crack for the stress τ_{yz}, respectively. The c_{L} is about equal to $c_{\mathrm{L}} \approx 0.2179$ for $b = 0.1$, $c_{\mathrm{L}} \approx 0.1906$ for $b = 0.2$, $c_{\mathrm{L}} \approx 0.1799$ for $b = 0.3$, $c_{\mathrm{L}} \approx 0.1742$ for $b = 0.4$, $c_{\mathrm{L}} \approx 0.1700$ for $b = 0.5$, respectively. The c_{R} is about equal to $c_{\mathrm{R}} \approx 0.1664$ for $b = 0.1$, $c_{\mathrm{R}} \approx 0.1613$ for $b = 0.2$, $c_{\mathrm{R}} \approx 0.1579$ for $b = 0.3$, $c_{\mathrm{R}} \approx 0.1543$ for $b = 0.4$, $c_{\mathrm{R}} \approx 0.1511$ for $b = 0.5$, respectively. Here, the parameter describing the functionally graded materials λ is equal to 0.2. The c_{R} and c_{L} decrease with decrease of the crack length, but they decrease slowly.

Acknowledgements

The authors are grateful for the financial support by the Natural Science Foundation of Hei Long Jiang Province (A0301), the Natural Science Foundation with Excellent Yong Investigators of Hei Long Jiang Province (CJ04-08) and the National Natural Science Foundation of China (50232030,10172030).

References

Amemiya, A., Taguchi, T., 1969. Numerical Analysis and Fortran. Maruzen, Tokyo.

Bao, G., Cai, H., 1997. Delamination cracking in functionally graded coating/metal substrate systems. Acta Materialia 45, 1055–1066.

Chen, Y.F., 1990. Interface crack in nonhomogeneous bonded materials of finite thickness. Ph.D. Dissertation, Lehigh University.

Delale, F., Erdogan, F., 1988. On the mechanical modeling of the interfacial region in bonded half-planes. ASME Journal of Applied Mechanics 55, 317–324.

Edelen, D.G.B., 1976. Nonlocal field theory. In: Eringen, A.C. (Ed.), Continuum Physics, vol. 4. Academic Press, New York, pp. 75–204.
Erdelyi, A. (Ed.), 1954. Tables of Integral Transforms, vol. 1. McGraw-Hill, New York.
Erdogan, F., Wu, H.B., 1996. Crack problems in FGM layer under thermal stress. Journal of Thermal Stress 19, 237–265.
Erdogan, F., Wu, B.H., 1997. The surface crack problem for a plate with functionally graded properties. Journal of Applied mechanics 64, 449–456.
Eringen, A.C., 1972. Linear theory of nonlocal elasticity and dispersion of plane waves. International Journal of Engineering Science 10, 425–435.
Eringen, A.C., 1976. Nonlocal polar field theory. In: Eringen, A.C. (Ed.), Continuum Physics, vol. 4. Academic Press, New York, pp. 205–267.
Eringen, A.C., 1977. Continuum mechanics at the atomic scale. Crystal Lattice Defects 7, 109–130.
Eringen, A.C., 1978. Linear crack subject to shear. International Journal of Fracture 14, 367–379.
Eringen, A.C., 1979. Linear crack subject to anti-plane shear. Engineering Fracture Mechanics 12, 211–219.
Eringen, A.C., 1983. Interaction of a dislocation with a crack. Journal of Applied Physics 54, 6811–6817.
Eringen, A.C., Kim, B.S., 1974. On the problem of crack in nonlocal elasticity. In: Thoft-Christensen, P. (Ed.), Continuum Mechanics Aspects of Geodynamics and Rock Fracture Mechanics. Reidel, Dordrecht, Holland, pp. 81–113.
Eringen, A.C., Kim, B.S., 1977. Relation between nonlocal elasticity and lattice dynamics. Crystal Lattice Defects 7, 51–57.
Eringen, A.C., Speziale, C.G., Kim, B.S., 1977. Crack tip problem in nonlocal elasticity. Journal of the Mechanics and Physics of Solids 25, 339–346.
Forest, B., 1998. Modeling slip, kind and shear banding in classical and generalized single crystal plasticity. Acta Materialia 46, 3265–3281.
Gradshteyn, I.S., Ryzhik, I.M., 1980. Table of Integrals, Series and Products. Academic Press, New York, p. 480.
Green, A.E., Rivilin, R.S., 1965. Multipolar continuum mechanics: Functional theory I. Proceeding of The Royal Society of London A 284, 303–315.
Itou, S., 1978. Three dimensional waves propagation in a cracked elastic solid. ASME Journal of Applied Mechanics 45, 807–811.
Itou, S., 2001. Stress intensity factors around a crack in a non-homogeneous interface layer between two dissimilar elastic half-planes. International Journal of Fracture 110, 123–135.
Jin, Z.H., Batra, R.C., 1996. Interface cracking between functionally graded coating and a substrate under antiplane shear. International Journal of Engineering Science 34, 1705–1716.
Morse, P.M., Feshbach, H., 1958. Methods of Theoretical Physics. McGraw-Hill, New York, p. 926.
Nowinski, J.L., 1984. On nonlocal aspects of the propagation of Love waves. International Journal of Engineering Science 22, 383–392.
Ozturk, M., Erdogan, F., 1996. Axisymmetric crack problem in bonded materials with a graded interfacial region. International Journal of Solids and Structures 33, 193–219.
Pan, K.L., 1992. The image force on a dislocation near an elliptic hole in nonlocal elasticity. Archive of Applied Mechanics 62, 557–564.
Pan, K.L., 1994. The image force theorem for a screw dislocation near a crack in nonlocal elasticity. Journal of Applied Physics 27, 344–346.
Pan, K.L., 1995. Interaction of a dislocation with a surface crack in nonlocal elasticity. International Journal of Fracture 69, 307–318.
Pan, K.L., 1996. Interaction of a dislocation and an inclusion in nonlocal elasticity. International Journal of Engineering Science 34, 1657–1688.
Pan, K.L., Fang, J., 1993. Nonlocal interaction of dislocation with a crack. Archive of Applied Mechanics 64, 44–51.
Pan, K.L., Fang, J., 1996. Interaction energy of dislocation and point defect in bcc iron. Radiation Effect Defects 139, 147–154.
Pan, K.L., Takeda, N., 1997. Stress distribution on bi-material interface in nonlocal elasticity. In: Proceeding of the 39th JSASS/JSME Structure Conference, Osaka, Japan, pp. 181–184.
Pan, K.L., Takeda, N., 1998. Nonlocal stress field of interface dislocations. Archive of Applied Mechanics 68, 179–184.
Pan, K.L., Xing, J., 1997. On presentation of the boundary condition in nonlocal elasticity. Mechanics Research Communications 24, 325–330.
Rice, J.R., 1968. A path independent integral and the approximate analysis of strain concentrations by notches and cracks. ASME Journal of Applied Mechanics 35, 379–386.
Shbeeb, N.I., Binienda, W.K., 1999. Analysis of an interface crack for a functionally graded strip sandwiched between two homogeneous layers of finite thickness. Engineering Fracture Mechanics 64, 693–720.
Sun, Y.G., Zhou, Z.G., 2004. Stress field near the crack tip in nonlocal anisotropic elasticity. European Journal of Mechanics A/Solids 23 (2), 259–269.
Xia, Z.C., Hutchinson, J.W., 1996. Crack tip fields in strain gradient plasticity. Journal of the Mechanics and Physics of Solids 44, 1621–1648.
Yan, W.F., 1967. Axisymmetric slipless indentation of an infinite elastic cylinder. SIAM Journal of Applied Mathematics 15, 219–227.

Zhou, Z.G., Shen, Y.P., 1999. Investigation of the scattering of harmonic shear waves by two collinear cracks using the nonlocal theory. Acta Mechanica 135, 169–179.
Zhou, Z.G., Wang, B., 2003. Investigation of anti-plane shear behavior of two collinear impermeable cracks in the piezoelectric materials by using the nonlocal theory. International Journal of Solids and Structures 39, 1731–1742.
Zhou, Z.G., Han, J.C., Du, S.Y., 1999. Investigation of a Griffith crack subject to anti-plane shear by using the nonlocal theory. International Journal of Solids and Structures 36, 3891–3901.
Zhou, Z.G., Wang, B., Du, S.Y., 2003. Investigation of anti-plane shear behavior of two collinear permeable cracks in a piezoelectric material by using the nonlocal theory. ASME Journal of Applied Mechanics 69, 388–390.